is a result to have arrived at of the very first order of importance.

I have next in connection with that diagram to give another which we owe to the labours of a German observer, Prof. Spörer. Not only have we to accept the fact that these important solar phenomena are limited to certain zones, but we have to study that fact in connection with another, that though all of them vary very violently, they all have what is called a cycle, and the cycle affects the particular zone of the sun on which they appear. Here a sunspot curve, as it is called, writes out for us in a graphic form the quantity of spotted area on the sun from year to year. It begins at 1867, and ends at 1878. This curve means that when the curve is at its highest, we get the greatest number of spots, or the greatest amount of spotted area on the sun's surface. At one place we get a very sudden increase of the spotted area. The curve is almost like a chalk cliff, it goes straight up, but it does not come so straight down. The curve from the minimum of spots on the sun to the maximum is very much steeper than that from the maximum to the next minimum. The sunspot period on an average is one of about eleven years, and it may be said, though I do not want the term to be misunderstood, to represent the seasons on the sun, because when we get that curve low, we see the sun for days together without any spots on it at all. When we get to the highest part, of course there is the greatest number of spots on it.

In connection with that period then, which, as it is good for spots, must also be good for faculæ and for metallic protuberances, after the results obtained by the Italian observers, it is most interesting to see in further detail whether there is any difference in the part of the sun thus affected. The two lower curves show us that when there is the smallest number of spots on the sun—when there is a sunspot minimum—the spots that appear are seen in a high latitude, and that latitude goes on decreasing and decreasing regularly and gradually until we get, at the next minimum, a real over-lapping of two perfectly distinct spotted areas. When we have the maximum period of sunspots, the latitude of the sunspot zone is between 8° and 10°, but it gets much lower than that when the period is closing, and even before one period has closed another one has begun in a higher latitude, so that the swirl in the solar atmosphere seems to begin in a high latitude—say 30° or 35°, or thereabouts—and very soon gets into full swing in latitude between 10° and 12°, and then it very gradually dies away until spots and metallic prominences and faculæ cling pretty near to the equator, and then we get a new wave of activity, beginning again in a high latitude, as is indicated by the beginning of the second curve.

Drawings made by Mr. Carrington a good many years ago show this result in another form, which emphasises the enormous difference in the amount of spotted area, as it is called, at the maximum and minimum time. Another diagram gives the results of the last eleven years' work at Greenwich, where they have been computing the positions of the spots obtained on their photographs and on the photographs which the Solar Physics Committee receives from India. This gives the history of the sunspot period in rather a different way; we begin in the year 1873, and end in the year 1884, and the curves represent the amount of faculæ, of penumbræ, and umbræ. Here again we get both faculæ, penumbræ, and umbræ increasing towards the maximum period, and it is seen that when we come to discuss photographs instead of depending on eye-observations, as the Italians did, we still find that the faculæ and the spots vary together. Another diagram shows another important matter. We are now discussing at Kensington the results obtained from the photographs from several points of view. One point of view is this. It seemed hard, after all the trouble taken to observe latitudes, that all spots north and south of the equator should be lumped together in a mere statement of spotted areas. The two upper curves in the diagram represent the spots north and the spots south; and an important thing which comes out of this is that the curve representing the greatest amount of spotted area north and that representing the greatest amount of spot area north and south do not go together. We do not get the greatest amount of spots north and south of the equator at the same time. A peak in the south curve is in two or three cases associated with a valley in the north curve.

J. NORMAN LOCKYER

(*To be continued.*)

THE CORRELATION OF THE DIFFERENT BRANCHES OF ELEMENTARY MATHEMATICS[1]

AMONG the permanent acquisitions to mathematical science secured within the last half century, within the limits of those branches with which our Association concerns itself, two (I conceive) stand out as pre-eminent in their far-reaching and all-pervading consequences.

These are the firm establishment as distinct entities of two concepts, which have been fixed for all the future of science in the terms *Energy* and *Vector*, and the development of the groups of ideas and principles which cluster around each.

The term *Energy* indeed, and the great principle of the *Conservation*, or (as I prefer with H. Spencer to call it) *Persistence*, of Energy, the establishment of which will live in the history of science as the great achievement of the central part of the nineteenth century, have a scope far beyond the purely mathematical treatment of dynamics and the allied branches of physical sciences. They, the concept and the principle, have already profoundly modified the views of the physicist as to the natural laws with which he is concerned, and are destined to form the starting-point and firm foundation for all his conquests in the future. But no less is it true that the conception of energy, while it has naturally arisen out of the higher mathematical treatment of dynamics, has necessitated a very material recasting of that treatment in its most elementary, as well as in its more advanced, stages, if it is to bear any fruitful relation to physical science in general. This recasting of elementary dynamics, if not yet fully and satisfactorily effected in most of the text-books which still remain in use, in which the notion of energy is brought in rather as the "purple patch" than inwoven into the whole texture of the robe in which the subject is clothed, is yet, thanks pre-eminently to the teaching of Maxwell, Thomson and Tait, and Clifford, in a fair way for being accomplished.

The influence of the conception of energy is, however, as regards mathematics, rather an influence from without than one from within its peculiar domain.

That which is strictly mathematical in the treatment of any science is not its subject-matter, but the *form* in which that subject-matter must from its nature be expressed. Mathematics, as such, is in fact *a formal* (may I not say *the* formal?) science, concerning itself with the particular matter only so far as that matter necessitates a particular form for its expression. Hence the recurrence of the same formulæ and mathematically the same propositions in different branches of science, so that, to take elementary instances, a proposition in geometry may be read off as a proposition in statics by substituting forces for lines, or the formula which determines the speed of the centre of mass of two masses having different speeds is also that which determines the temperature resulting from the mixture of two masses of different temperatures.

To this *formal*, or essentially mathematical, part of the exact sciences belongs the conception of a *Vector*, or rather the group of conceptions which cluster around that term. The term itself was introduced by Hamilton in connection with his grand theory of quaternions about forty years since, but the idea had been already firmly grasped and developed so as to afford a complete explanation of the imaginary ($\sqrt{-1}$) of ordinary algebra within the twenty years preceding that epoch. In fact in the year 1845 I myself enjoyed the privilege, as a young student, of attending lectures of De Morgan on this subject, which he afterwards developed in his treatise on "Double Algebra," published in 1849.[2] I think, however, that we may conveniently date from the introduction of the term "*Vector*," which is now the accepted term for any magnitude which besides numerical quantity or intensity has a definite direction in space, the definitive acquisition of this concept with all its consequences to the settled territory of mathematical science. The calculus of quaternions indeed, or that part of it which was truly original and due to the genius of Hamilton alone, involving the conceptions of the products and quotients of vectors in three-dimensional space, is doubtless beyond the range of what now can be, or within the near future is likely to be, regarded as elementary mathematics; but the notions of vector addition and subtraction and their consequences in geometry and mechanics ought assuredly to be considered as within that range, as ought also, for a complete view of ordinary algebra, vector products and quotients in one plane.

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[1] A paper read before the Association for the Improvement of Geometrical Teaching by the President, R. B. Hayward, F.R.S. (see NATURE for January 21, p. 277). We print the address in the hope that a discussion of some of its principles may ensue.

[2] Sir W. R. Hamilton's Lectures on Quaternions were published in 1853.

If we inquire in what manner we should expect the idea of a Vector and its attendant ideas to affect our elementary teaching, I think the answer would be that it would naturally lead to a different grouping, or arrangement in order, of the various branches taught. It would lead us to group them not according to their subject-matter—arithmetic and algebra, the sciences of number, particular and general; geometry, the science of space; trigonometry, in one aspect treating of space and number in combination, in another as a development of algebra; statics, dealing with forces in equilibrium; dynamics, with forces producing motion; and so on—but according to their *form*, as dependent on the nature of the magnitudes dealt with. One-dimensional magnitudes, that is, magnitudes defined by one element only, whether such as are completely defined by one element, or more complex magnitudes regarded for the moment in respect of one of their elements only, would naturally form the first stage, with subdivisions according as the treatment is purely *quantitative* or *metric*, or *scalar*, that is, metric with the addition of the notion of sign or sense. Then would follow two-dimensional magnitudes, or magnitudes defined by two elements, treated with respect to both elements in subdivisions *metric* and *scalar* as in the first stage, and also finally as complete *Vectors*.

If we further inquire how far these notions have in fact affected our elementary text-books, I think we shall find that the extent to which they have done so is very small. A comparison of the text books of the present day (I speak of them in the gross, not forgetting that there are some important exceptions) with those that were current at the time when my own mathematical studies began, an interval of some forty years, produces the impression of likeness rather than that of contrast. Changes, which are welcome improvements, have doubtless been made in matters of detail, and in various ways the paths have been smoothed for the student; but the general treatment is essentially the same, and shows very little sign of independent thought, informed by more extended views, having been exercised with regard to the old traditional modes of presenting the subject as a whole.

The algebra, for instance, of our ordinary text-books is (if I may venture to give it a nickname which every brother Johnian at any rate will understand) *heptadiabolic*,[1] or that whose highest outcome, in the mind of the pupil who has studied it, is the solution (so called) of a hard equation or equation problem—little more in fact than a series of rules of operation, which skilfully used (and how many fail to attain even this amount of skill) will solve a few puzzles at the end, but very barren of any intellectual result in the way of mental training:—an algebra in which the interpretation of negative results and the use of the negative sign as a sign of affection has been ignored, or so lightly dwelt on, that the notion of the signs + and − as appropriately expressing opposite senses along a line, has to be elaborately explained as almost a new idea in commencing trigonometry; and further, an algebra which, as Prof. Chrystal has observed in his address to the British Association, is useless as an instrument for application to co-ordinate geometry, so that the student has at this stage practically to study the subject again, and only then obtains something of a true notion of what algebra really is.

With the foregoing general considerations as a guide, I will now examine in some detail the correlation or affiliation of the several branches of elementary mathematics to which they seem to lead.

Mathematics naturally begins by treating magnitudes with reference to the single element of quantity. The answers to the simple questions, How many? How much? How much greater? How many times greater? lead up to the arithmetic of abstract and concrete number, and the doctrines of ratio and proportion, and the development of these with the use of the signs +, −, &c., as signs of the elementary operations, and letters to denote numbers or ratios, naturally leads to generalised arithmetic or arithmetical algebra. At this stage $a - b$, where b is greater than a, is an impossible quantity, and a negative quantity has by itself no meaning. In this earliest stage the magnitudes dealt with are either pure numbers or concrete one-dimensional magnitudes, value, time, length, weight, &c., whose measurements are assumed as known. There are few magnitudes which are metric or quantitative *only*, but all magnitudes have quantitative relations which may be regarded apart from their other relations, and so may be the matter or subject of arithmetic, if they are such that their quantity can be estimated definitely or measured. Purely metric magnitudes are such as can be conceived to be reduced in quantity down to zero or annihilated, but of which the negative is inconceivable, so that at zero the process must stop. Such are many magnitudes that are measured by integral numbers—as population, numbers of an army or a flock, a pile of shot, &c., or continuous magnitudes, such as mass, energy, quantity of heat or light, the moisture of the atmosphere, the saltness of water, &c. But there is a far larger class of magnitudes, of which it is true that not only the opposite or negative can be conceived, but that they cannot be fully treated without regard to such opposite. For these, reduction to zero, or annihilation, is only a stage in passing from the magnitude to its opposite, *e.g.* time after and time before a given epoch, lengths forward and backward along a line, receipts and payments, gain and loss, and so forth. The consideration of such magnitudes at once leads to the scalar subdivision of the one-dimensional stage. In this, magnitudes which are themselves purely scalar, or the scalar elements of more complex magnitudes, are alone considered. But to the quantitative element is now superadded the notion of sign or sense, appropriately denoted by the signs + and −, which, without ceasing to be signs of operation, are now regarded also as signs of affection. The introduction of this notion leads at once to scalar algebra, in which $a - b$, where b is greater than a, is no longer an impossible quantity, and a negative result has a definite meaning, so long as the magnitudes dealt with are not purely metric. The step from arithmetical to scalar algebra, though very simple and almost insensibly made, should, I think, be much more distinctly emphasised in our teaching and our text-books than is usually the case. Exercises in metric and scalar readings of the same simple expressions should be frequent, and negative results, whenever they occur, examined and shown to be impossible only if the magnitude in the question is purely metric, but interpretable if it is scalar. Thus the idea would be gradually evolved that the impossibles or imaginaries of algebra are so in a purely relative sense and with regard to the particular subject-matter treated of, and it would become readily conceivable that the remaining impossible quantity $a + b\sqrt{-1}$, to which form scalar algebra, working on the basis of its laws of combination, would show that all expressions may be reduced, may be completely interpretable when ultra-scalar magnitudes form the subject of investigation.

Passing now to the consideration of special magnitudes and how far their discussion can be carried in the one-dimensional stage, I think we shall arrive at some important practical results.

The scalar element of space is length measured forwards or backwards along a line, and the resulting geometry is the simple geometry of points on the same line. Starting from the definition that $-AB$ is BA, the fundamental proposition is that $AB + BC = AC$, whatever be the positions of A, B, C on the line, and this with a few simple consequences completes all that is necessary to be considered in linear geometry.

Combine with this the notion of time, and the science of linear or scalar kinematics emerges This includes the measurement of the motion of a point along a given line by the scalar magnitudes, *speed*[1] and *acceleration*, and the discussion of different kinds of linear motion, uniform and uniformly accelerated, and so the laws of falling bodies. When the notion of a variable rate became firmly grasped, the investigation might be extended to some simple cases of variable acceleration without any large demand on algebraical skill, and so the fundamental notions of the fluxional or differential calculus and some idea of its scope and aim be attained.

Introduce now the notions of force and mass and the axioms of force or motion as contained in Newton's laws, and the science of linear or scalar dynamics results. If we drop for an instant the notion of time, or rather of *change in time*, we have linear statics, which consists of little more than the single proposition—the "tug-of war" proposition—that the resultant of any number of forces along the same line is their scalar sum. Linear kinetics, however, covers a wide field—the relations of force, mass, and acceleration, their measures and their applications to simple cases of linear motion; the time integral of force, momentum; the space integral, work; energy, kinetic and potential; the relations of force applied to resistance overcome in simple machines working steadily; impulsive actions in collisions and explosions; and other simple developments—would here be studied in their simplest forms apart from any greater mathematical difficulties than arithmetic and very rudimentary algebra, and yet involving almost every truly dynamical principle needed for the highest problems in dynamics. Here, with perhaps a few applications to other branches of physics, the range of the one-dimensional stage ends.

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[1] The allusion is to a paper which used to be set at the annual May Examinations at St. John's College, Cambridge, consisting of seven very hard equations and equation problems, familiarly known as the "seven devils." As a test of a certain kind of skill in algebraical operation and of ingenuity and clearheadedness it was not without considerable value, but it tended to produce false notions of algebra in its relations to mathematics generally.

[1] The term *speed* has been happily appropriated for the scalar element of velocity. A corresponding term is wanted for the scalar element of acceleration; no better word than *quickening* suggests itself to me.

Proceeding next to two-dimensional magnitudes, we commence of course with elementary plane geometry, in which the propositions, which are not purely descriptive, deal with the magnitudes considered in purely metric relations.

The introduction of the notion of sense for lines and angles, denoted by the signs + and −, leads in one direction to elementary trigonometry, and in another to co-ordinate geometry.

Kinematics is now extended to motion in two dimensions, and this should lead at once to the notion of velocity, acceleration, &c., as vector magnitudes, and with this the general notion of a vector and vector addition. In dynamics force emerges as a vector, and the composition of forces regarded as the addition of vectors lays the foundation of statics, or the relations of forces independent of the element of time, to be developed on the one side with the aid of pure geometry and graphical methods, on the other by the application of trigonometry. This is naturally succeeded by uniplanar kinetics, developed more or less fully till it extends to regions beyond the range of elementary mathematics. Algebra will have been carried on *pari passu* to meet the increasing requirements of the special subjects, but will still remain scalar with its impossible or uninterpreted symbols.

The next step is to complete the algebra of vectors in one plane. The notion of a vector and vector addition will already have been grasped and will need only some further application and development, but the extension of the notion of multiplication to vectors in one plane at once leads to the already familiar algebra, but with wider meaning and without impossible quantities or uninterpretable symbols. The immediate result is a complete trigonometry, of which De Moivre's theorem, now completely intelligible and not a mere formula, forms the basis, and the higher developments of ordinary elementary algebra. It will then appear that ordinary algebra receives its full explanation in vectors limited to one plane, and it will naturally be anticipated that the algebra of vectors in any directions in three-dimensional space will be different from the ordinary algebra, an expectation which will be amply justified by the study of the algebra or calculus of quaternions, the grand discovery of Sir W. R. Hamilton, but to pass on to this would be to pass beyond the limits of what is, in the sense of our Association, elementary mathematics.

If the correlation of the elementary branches of mathematics, which I have now sketched out, is accepted as based on true principles, I cannot doubt but that it will lead to important practical consequences, the development of which I may safely leave in the hands of those who so accept it. There are, however, a few immediate deductions from it, which occur to me as naturally calling for expression before I close this paper.

In the first place I would observe that while I believe the several stages in the foregoing scheme to be natural and such as every teacher would do well to have in his own mind in arranging the course of instruction for his pupils, I do not at all regard it as marking out the exact order to be followed by each individual student. There is room here, still in subordination to the general scheme, for wide variation according to the different requirements of different students. It would in almost all cases, I think, be very unwise that any one of the stages should be completed before the next was commenced. For instance, though the theory of ratio is purely one-dimensional and metric, no one, I suppose, would think of dealing with it otherwise than in the incomplete form sufficient for arithmetic before commencing the study of the simple two-dimensional geometry of Euclid or our own text-book. And again, how far scalar or linear kinematics and dynamics should be studied (or whether at all) before proceeding further in the two-dimensional stage to trigonometry, &c., is a question which may fairly be answered in different ways according to the different objects aimed at in the study of mathematics by different classes of students.

It appears to me too to follow from our scheme that, whatever may be true for the select few who aim at becoming mathematicians, for the great mass of those with whom the chief object is, or ought to be, intellectual training, algebra should be studied at first, not as a subject for its own sake, but as an instrument for use in other subjects. I hold that, unless pursued into its higher developments, algebra *per se* is not a valuable instrument of mental training. Can it be said that such algebra as is required (say) at the Previous Examination at Cambridge, a large part of which has had no application for the student in any other subject, is of any value at all proportional to the time it has taken him to acquire it? I think, then, that algebra should be studied piecemeal: first just that small quantity which is necessary for one-dimensional magnitudes treated as scalars; then, when the need was felt from the occurrence of problems requiring more knowledge of algebra, adding more, and so on continually, keeping up the study of algebra concurrently with, and only slightly in advance of, the requirements of the subjects to which it is applied.

Again, our scheme suggests, I think, a definite answer to the question:—What minimum of mathematics is it reasonable to expect every educated man, not professing to be a mathematician, to have acquired?

I think there are few who are satisfied with the answer practically given to this question by our Universities in their first examinations for matriculation or degrees. At Cambridge, the question with reference to the "Little Go" Examination is even now under consideration. I would submit that the subjects included within our one-dimensional scalar stage together with elementary geometry, and statics, treated geometrically, or by graphical methods only, from the two-dimensional, would constitute a far more satisfactory minimum than the present. This would exclude a good deal of the algebra now expected and the trigonometry, but would add linear kinematics and kinetics. The student, who had gone through such a course, would not probably be able to effect any but the simpler algebraical reductions or solve any but the simplest kinds of equations; but he would have gained some notion of what an algebraical formula means as the expression of a law, and be able to deduce from it numerical consequences and to follow out the simpler general results obtainable from it, and he would have acquired a clearer conception and higher appreciation than is common with people otherwise well educated of the part which mathematics plays in its application to the physical sciences, and with it that sound dynamical basis which is the essential condition of a fruitful study of physics. I feel sure, too, that the consciousness of the student that he was dealing with actual living laws and not with the dry bones of algebraical processes or trigonometrical formulæ leading to no results, and that his mathematical studies were meant to be, and were, more than a mere mental gymnastic, would add life and interest to those studies which would react on the whole of his mental training.

I may note, further, that our scheme seems to give the best answer to the question which has frequently been mooted of late, in our Association as well as elsewhere, whether statics should precede kinetics, or whether it should be treated as a particular case of the more general science. Linear kinematics and kinetics, being one-dimensional and scalar, may well precede the study of statics, which deals with vectors, though not of necessity in the case of one who has attained sufficient knowledge of elementary geometry not to be stopped by mere geometrical difficulties; but vector or uniplanar kinetics, on account of its much greater complexity and its consequent larger demands on mathematical attainments, would in general naturally follow a somewhat detailed study of statics.

I will take this opportunity of making one other remark, which, if it does not directly arise out of the present discussion, is closely akin to it, and that is on the importance of our teaching of the several branches of mathematics being *proleptic*, or looking forward in one stage to what will be required in a higher stage. In definitions for instance, of two that are equally good for the immediate purpose, that one is to be preferred which will be intelligible and useful when the term defined comes to be extended to higher matter.

Thus I conceive that multiplication should be defined from the outset in such a manner as would make it applicable to a fractional as well as to an integral multiplier. If I explain that to multiply 6 by 5 is to repeat 6 five times and find the aggregate result, my explanation fails when I am asked to multiply 6 by $\frac{3}{4}$; but if I use De Morgan's definition that "multiplication consists in doing with the multiplicand what is done with the unit to form the multiplier," or an explanation of multiplication cast in this form, I have given an explanation equally simple with the former and applicable also to a fractional multiplier.

Again, in the very beginning of arithmetic, which is counting, I maintain that much would be gained if from the first the child were taught to count, not one, two, &c., but *nought, nothing*, or *zero*, one, two, three, &c.; and then if, later on, ordinal reckoning were made to accord with this, though here unfortunately language and usage fails to supply the word wanted, for which, for want of a better, I must coin the form *zeroth* (*noughtth* or *nothingth* being out of the question), thus: *zeroth*, first, second, &c. Then the transition to counting below zero by negative numbers would follow at once as by a natural development, when the need for it arose. Thus when it came to the notation of numbers, the place of a digit would properly be reckoned from the units as the *zeroth* place (not the *first*), and would be extended naturally by negative ordinal reckoning downwards, when decimal fractions are introduced.

This leads me to another illustration, which I am also anxious to introduce as a suggestion on its own merits. Prof. Chrystal has complained that to many students even when beginning co-ordinate geometry the idea of the *order* of a term or an expression is unfamiliar. Now it has occurred to me that this is just the word wanted, to replace the five-syllable word "characteristic," which has been used (or perhaps has *not* been used just because it is pentasyllabic) to express the distance of any digit of a number in order from the unit's digit. Let us speak of the unit's digit as of the order 0; the tens, hundreds, &c., digits of the orders 1, 2, &c.; and the tenths, hundredths, &c., of the orders -1, -2, &c.; and add to this that a number is said to be of the same *order* as that of its highest significant digit, and we have a language not only of the utmost use and importance in decimal arithmetic, but also at once applicable by the most natural extension to an algebraical expression arranged according to the powers of some letter or letters, while it would enable us conveniently to express in language numbers which transcend our ordinary numerical vocabulary, so that, for instance, 53×10^{12} might be read as 53 of the 12th order, and 53×10^{-12} as 53 of the -12th order, and so on.

In conclusion I will only add that, if in this paper I have in any parts expressed myself somewhat dogmatically, I have done so in the hope of challenging discussion, and only claim the acceptance of the views which I have tried to express *distinctly*, if *briefly*, in the event of discussion resulting in a verdict in their favour.

ON THE METHOD OF STATING RESULTS OF WATER ANALYSES

THE Chemical Society of Washington is desirous of bringing before chemists and others interested, the report of Committee herewith inclosed; and as NATURE has a wide circulation in this country, I am authorised to send a copy of the abstract to your journal, hoping you may find space for it.

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Sec. Chem. Soc. Washington
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Washington, D.C., February 25

The Chemical Society of Washington, at the meeting of November 12, 1885, appointed a Committee to consider the present state of water analysis, and to present a method of stating analyses, adapted for general use, in order that those hereafter published may be readily compared with each other and with future work. This Committee reported February 11, 1886, and was authorised to prepare an abstract for publication, in order to call the attention of chemists to the subject. The Society earnestly recommends the adoption of the scheme which is herewith briefly presented. The full text of the report will be published in the next *Bulletin* of the Society.

(*Abstract*)

Water analyses are usually made to answer one of three questions, viz:—

(1) Is the water useful medicinally?
(2) Is it injurious to health? and
(3) Is it suitable for manufacturing purposes?

Many books relating to water were published during the eighteenth century, but accurate chemical analysis was not attempted until about 1820. As the earlier analyses were isolated, rare, and made for special purposes, the form of the statement was of little importance if it were only intelligible. At the present time, however, water-analyses are very numerous. An examination of about a thousand shows some forty two methods of stating quantitative results, there being sometimes three different ratios in the report of one analysis. Such discrepancies render comparisons difficult and laborious. The various methods of statement may be classified under the following general forms:—

(1) Grains per imperial gallon of 10 lbs. or 70,000 grains.
(2) Grains per U.S. or wine gallon of 58,372+ grains.
(3) Decimally, as parts per 100, 1000, 100,000, or 1,000,000.
(4) As so many grammes or milligrammes per litre.

The last two would be identical if all waters had the same density; but as the densities of sea water, mineral waters, &c., are much above that of pure water, it is plain that the third and fourth modes are not comparable. The Committee therefore unanimously recommends—

(1) That water-analyses be uniformly reported according to the decimal system, in parts per million or milligrames per kilogramme, with the temperature stated, and that Clark's scale of degrees of hardness and all other systems be abandoned.

(2) That all analyses be stated in terms of the radicals found.

(3) That the constituent radicals be arranged in the order of the usual electro chemical series, the positive radicals first.

(4) That the combination deemed most probable by the chemist should be stated in symbols, as well as by name.

The abandonment of Clark's *scale* has been recommended by Wanklyn and Chapman; and the recommendation made by the Committee does not involve the disuse of his method, but merely the bringing it into accord with the decimal system—the changing from grains per gallon to milligrammes per kilogramme.

The last conclusion (4) was deemed desirable from the frequent confusion in the statement of the iron salts and of the carbon oxides.

The Committee is unanimously of the opinion that analyses in the form recommended will prove quite as acceptable to Boards of Health and to the public in general, for whom such analyses are often made, as if presented in the mixed and irregular forms commonly adopted.

The Committee also feels sure that the people in general are better able to form a definite idea of the character of a water from a report stated in parts per 100, parts per 1,000,000, &c., than from one expressed as grains per gallon, the latter being a ratio wholly unfamiliar to any but those in the medical or pharmaceutical professions.

(Signed) A. C. PEALE, M.D.
WM. H. SEAMAN, M.D.
CHAS. H. WHITE, M.D.

ON SOME POINTS IN THE PHYLOGENY OF THE TUNICATA

IN his monograph on the genus *Doliolum*, forming one of the recent parts of the series illustrating the Fauna and Flora of the Bay of Naples,[1] Dr. B. Uljanin gives a sketch of the phylogeny of the Tunicata, some parts of which cannot, I think, be accepted without considerable modification. Uljanin has evidently regarded the subject from the *Doliolum* point of view, and, in fact, he only introduces the other groups of Tunicata for the purpose of discussing their relationships to *Doliolum*. Consequently it is not to be wondered at that he should assign rather too central a position to that genus, and give it too much importance relatively to the other groups. What is of more importance is that his scheme shows a course of evolution which seems not altogether in accordance with

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[1] "Fauna und Flora des Golfes von Neapel." X. Monographie: Doliolum, von Dr. Basilius Uljanin. (Leipzig, 1884.)